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(By appointment)
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Discrete mathematics is the study of mathematical structures that are countable or otherwise distinct and separable. Examples of structures that are discrete are combinations, graphs, and logical statements. Discrete structures can be finite or infinite.
Course Syllabus 课程大纲

Chapter 0 Introduction
Chapter 1 Set Theory
Chapter 2 Counting
Chapter 3 Binary Relation
Chapter 4 Special Relations
Chapter 5 Functions
Chapter 6 Graph Theory
Chapter 7 Trees
Chapter 8 Special Graphs
Course website: https://courses.gdut.edu.cn/course/view.php?id=489
0 Introduction

• Classroom:
  教2-209 （2# Teaching Building, RM 209）

• Online Teaching (Before School Openings)
  Tencent Meeting System

• Course Teaching Assistants (TAs)
  Mr. LIN, Yanjia
  Mr. LONG, Xin
  Dr. WU, Yalan
Score composition 课程评分

<table>
<thead>
<tr>
<th>Items</th>
<th>Scores percentage</th>
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<tr>
<td>Homework, Attendance</td>
<td>20-30%</td>
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<td>Student performance during classes</td>
<td>At least 5%</td>
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<td>Middle Term Exam</td>
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<td>End Term Exam</td>
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Tips for the course （课程小贴士）:

• Remember to prepare for the class
• Do homework by yourself
• Attend classes on time
• Be active during classes (bonus scores)
Goals (章节学习目标):
1 Knowing about the definition of Set (了解)
2 Knowing about the ways to describe Set （了解）
3 Understanding the operations of Set （理解）
4 Master the theorems of Set and their application （掌握）

Lay the foundation of function, probability analysis and so on
1.1 Sets (集合的定义)

• A set is an unordered collection of objects （No accurate definition）
  • the students in this class
  • the chairs in this room

• The objects in a set are called the elements, or members of the set. A set is said to contain its elements.

• $a \in A$ denotes that $a$ is an element of the set $A$.
• $a \notin A$ ?
Describing a set (集合的描述)  

Roster Method

\[ A = \{ a, b, c, d \} \]

\[ S = \{a,b,c,d\} = \{b,c,a,d\} = \{d,a,b,b,c\} \]

Ellipses (...) are used when the patterns are clear

\[ S = \{a,b,c,d, \ldots, z\} \]
Some Important Sets (重要集合)

\[\mathbb{N} = \text{natural numbers} = \{0,1,2,3,\ldots\}\]
\[\mathbb{Z} = \text{integers} = \{\ldots,-3,-2,-1,0,1,2,3,\ldots\}\]
\[\mathbb{Z}^+ = \text{positive integers} = \{1,2,3,\ldots\}\]
\[\mathbb{R} = \text{set of real numbers}\]
\[\mathbb{R}^+ = \text{set of positive real numbers}\]
\[\mathbb{C} = \text{set of complex numbers}\]
\[\mathbb{Q} = \text{set of rational numbers}\]
Describing a set (集合的描述)  

Set-Builder Notation

• Specify the property or properties that all members must satisfy:

\[ S = \{ x \mid x \text{ is a positive integer less than 100} \} \]
\[ O = \{ x \mid x \text{ is an odd positive integer less than 10} \} \]
\[ O = \{ x \in \mathbb{Z}^+ \mid x \text{ is odd and } x < 10 \} \]

• A predicate may be used:

\[ S = \{ x \mid P(x) \} \]

• Example: \( S = \{ x \mid \text{Prime}(x) \} \)

• Positive rational numbers:

\[ Q^+ = \{ x \in \mathbb{R} \mid x = \frac{p}{q}, \text{ for some positive integers } p, q \} \]
1 Set Theory

Describing a set (集合的描述)  

Interval Notation

\[ [a, b] = \{ x \mid a \leq x \leq b \} \]
\[ [a, b) = \{ x \mid a \leq x < b \} \]
\[ (a, b] = \{ x \mid a < x \leq b \} \]
\[ (a, b) = \{ x \mid a < x < b \} \]

问题1：How can we name \((a, b]\) or \([a, b)\) ?

*closed interval* \([a, b]\)

*open interval* \((a, b)\)
Describing a set (集合的描述)  Venn Diagram Notation

- The **universal set** $U$ is the set containing everything currently under consideration.
  - Sometimes implicit
  - Sometimes explicitly stated.
  - Contents depend on the context.
- The empty set is the set with no elements. Symbolized $\emptyset$, but \{\} also used.
问题2:
How many ways to describe a set?
到此，我们学到了几种描述集合的方法？
Russell’s Paradox (罗素悖论)

• Let $S$ be the set of all sets which are not members of themselves. A paradox results from trying to answer the question “Is $S$ a member of itself?”

• Related Paradox:
  • Henry is a barber who shaves all people who do not shave themselves. A paradox results from trying to answer the question “Does Henry shave himself?”

Bertrand Russell (1872-1970)
Cambridge, UK
Nobel Prize Winner
1 Set Theory

History of Set Theory (集合论的历史)

Cantor 1845-1918
Bertrand Russell (1872-1970)

The third mathematical crisis (第三次数学危机)
Set theory created by Cantor is not perfect

问题 3 How does the third mathematical crisis been solved?
Some things to remember

• Sets can be elements of sets.

\[
\{\{1,2,3\}, a, \{b, c\}\}
\]

\[
\{\mathbb{N}, \mathbb{Z}, \mathbb{Q}, \mathbb{R}\}
\]

• The empty set is different from a set containing the empty set.

\[
\emptyset \neq \{\emptyset\}
\]
Relations between Sets (Set Equality) A=B

**Definition:** Two sets are *equal* if and only if they have the same elements.

- Therefore if A and B are sets, then A and B are equal if and only if \( \forall x (x \in A \iff x \in B) \)
- We write \( A = B \) if A and B are equal sets.

\[
\{1,3,5\} = \{3, 5, 1\}
\]
\[
\{1,5,5,5,3,3,1\} = \{1,3,5\}
\]
Relations between Sets  (Subsets)

**Definition:** The set $A$ is a *subset* of $B$, if and only if every element of $A$ is also an element of $B$.

- The notation $A \subseteq B$ is used to indicate that $A$ is a subset of the set $B$.
- $A \subseteq B$ holds if and only if $\forall x (x \in A \rightarrow x \in B)$ is true.
  1. Because $a \in \emptyset$ is always false, $\emptyset \subseteq S$, for every set $S$.
  2. Because $a \in S \rightarrow a \in S$, $S \subseteq S$, for every set $S$.

**Examples:**
1. The set of all computer science majors at your school is a subset of all students at your school.
2. The set of integers with squares less than 100 is not a subset of the set of nonnegative integers.
Example:

**Theorem**  The empty set is the subset of any sets

**Proof:** \( \emptyset \subseteq A \iff \forall x \ (x \in \emptyset \rightarrow x \in A) \iff T \)

**Reading**

Homework: Read related logic and proofs contents in Chapter 1 of your textbook to learn about the expression (if \( p \), then \( q \), \( p \rightarrow q \))
Another look at Equality of Sets

• Recall that two sets $A$ and $B$ are *equal*, denoted by $A = B$, iff

\[
\forall x (x \in A \leftrightarrow x \in B)
\]

• Using logical equivalences we have that $A = B$ iff

\[
\forall x [(x \in A \rightarrow x \in B) \land (x \in B \rightarrow x \in A)]
\]

• This is equivalent to

$A \subseteq B$ and $B \subseteq A$
Example:

**Corollary** The empty set is unique

**Proof:** If there exist $\emptyset_1$ and $\emptyset_2$, then $\emptyset_1 \subseteq \emptyset_2$ and $\emptyset_1 \subseteq \emptyset_2$, therefore $\emptyset_1 = \emptyset_2$.
Proper Subsets $A \subset B$

**Definition:** If $A \subseteq B$, but $A \not= B$, then we say $A$ is a *proper subset* of $B$, denoted by $A \subset B$. If $A \subset B$, then

$$\forall x (x \in A \rightarrow x \in B) \land \exists x (x \in B \land x \notin A)$$

is true.
Set Cardinality

**Definition:** If there are exactly \( n \) distinct elements in \( S \) where \( n \) is a nonnegative integer, we say that \( S \) is *finite*. Otherwise it is *infinite*.

**Definition:** The *cardinality* of a finite set \( A \), denoted by \(|A|\), is the number of (distinct) elements of \( A \).

**Examples:**
1. \(|\emptyset| = 0\)
2. Let \( S \) be the letters of the English alphabet. Then \(|S| = 26\)
3. \(|\{1,2,3\}| = 3\)
4. \(|\emptyset| = 1\)
5. The set of integers is infinite.
Power Sets

**Definition**: The set of all subsets of a set $A$, denoted $P(A)$, is called the *power set* of $A$.

**Example**: If $A = \{a, b\}$ then $P(A) = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$

- $P(\emptyset) = \{\emptyset\}$,
- $P(\{\emptyset\}) = \{\emptyset, \{\emptyset\}\}$
- $P(\{1, \{2, 3\}\}) = \{\emptyset, \{1\}, \{\{2, 3\}\}, \{1, \{2, 3\}\}\}$

If $|A| = n$, Then $|P(A)| = 2^n$
Tuples

• The ordered $n$-tuple $(a_1, a_2, \ldots, a_n)$ is the ordered collection.

• Two n-tuples are equal iff their corresponding elements are equal.

• 2-tuples are called ordered pairs.

• The ordered pairs $(a, b)$ and $(c, d)$ are equal if and only if $a = c$ and $b = d$. 
1.2 Operations of Sets (集合的运算)

**Union** \( A \cup B = \{ x \mid x \in A \lor x \in B \} \)

**Intersection** \( A \cap B = \{ x \mid x \in A \land x \in B \} \)

**Difference** \( A - B = \{ x \mid x \in A \land x \notin B \} \)

**Symmetric Difference** \( A \oplus B = (A - B) \cup (B - A) \)

\[ = (A \cup B) - (A \cap B) \]

**Complementation** \( \bar{A} = U - A \)
Union

- **Definition**: Let $A$ and $B$ be sets. The *union* of the sets $A$ and $B$, denoted by $A \cup B$, is the set:

$$\{x \mid x \in A \lor x \in B\}$$

- **Example**: What is $\{1,3,4\} \cup \{3, 4, 5\}$?

**Solution**: $\{1,3,4,5\}$
1 Set Theory

Intersection

- **Definition**: The *intersection* of sets $A$ and $B$, denoted by $A \cap B$, is

\[ \{ x \mid x \in A \land x \in B \} \]

- Note if the intersection is empty, then $A$ and $B$ are said to be *disjoint*.

- **Example**: What is? $\{1,2,3\} \cap \{3,4,5\}$ ?

  **Solution**: $\{3\}$

- **Example**: What is?

  $\{1,2,3\} \cap \{4,5,6\}$ ?

  **Solution**: $\emptyset$
Difference

• **Definition:** Let $A$ and $B$ be sets. The *difference* of $A$ and $B$, denoted by $A - B$, is the set containing the elements of $A$ that are not in $B$. The difference of $A$ and $B$ is also called the complement of $B$ with respect to $A$.

$$A - B = \{x \mid x \in A \land x \notin B\} = A \cap \overline{B}$$

Venn Diagram for $A - B$
Complement

**Definition:** If \( A \) is a set, then the complement of the \( A \) (with respect to \( U \)), denoted by \( \bar{A} \) is the set \( U - A \)

\[
\bar{A} = \{x \in U \mid x \notin A\}
\]

(The complement of \( A \) is sometimes denoted by \( A^c \).)

**Example:** If \( U \) is the positive integers less than 100, what is the complement of \( \{x \mid x > 70\} \)

**Solution:** \( \{x \mid x \leq 70\} \)
1 Set Theory

Review Questions

Example: \( U = \{0,1,2,3,4,5,6,7,8,9,10\} \) \( A = \{1,2,3,4,5\} \) \( B = \{4,5,6,7,8\} \)

1. \( A \cup B \)  
   Solution: \( \{1,2,3,4,5,6,7,8\} \)

2. \( A \cap B \)  
   Solution: \( \{4,5\} \)

3. \( \bar{A} \)  
   Solution: \( \{0,6,7,8,9,10\} \)

4. \( \bar{B} \)  
   Solution: \( \{0,1,2,3,9,10\} \)

5. \( A - B \)  
   Solution: \( \{1,2,3\} \)

6. \( B - A \)  
   Solution: \( \{6,7,8\} \)
Symmetric Difference

**Definition**: The *symmetric difference* of $A$ and $B$, denoted by $A \oplus B$ is the set

$$(A - B) \cup (B - A)$$

**Example**:

$U = \{0,1,2,3,4,5,6,7,8,9,10\}$

$A = \{1,2,3,4,5\}$  $B = \{4,5,6,7,8\}$

What is:

- **Solution**: $\{1,2,3,6,7,8\}$
Set Identities

• Identity laws
  \[ A \cup \emptyset = A \quad A \cap U = A \]

• Domination laws
  \[ A \cup U = U \quad A \cap \emptyset = \emptyset \]

• Idempotent laws
  \[ A \cup A = A \quad A \cap A = A \]

• Complementation law
  \[ \overline{(\overline{A})} = A \]
Set Identities (Continued)

• Commutative laws

\[ A \cup B = B \cup A \quad A \cap B = B \cap A \]

• Associative laws

\[ A \cup (B \cup C) = (A \cup B) \cup C \]
\[ A \cap (B \cap C) = (A \cap B) \cap C \]

• Distributive laws

\[ A \cap (B \cup C) = (A \cap B) \cup (A \cap C) \]
\[ A \cup (B \cap C) = (A \cup B) \cap (A \cup C) \]
1 Set Theory

Set Identities (Continued)

• De Morgan’s laws
  \[ \overline{A \cup B} = \overline{A} \cap \overline{B} \quad \overline{A \cap B} = \overline{A} \cup \overline{B} \]

• Absorption laws
  \[ A \cup (A \cap B) = A \quad A \cap (A \cup B) = A \]

• Complement laws
  \[ A \cup \overline{A} = U \quad A \cap \overline{A} = \emptyset \]
Proving Set Identities

• Different ways to prove set identities:
  1. Prove that each set (side of the identity) is a subset of the other.
  2. Use set builder notation and propositional logic.
  3. Membership Tables: Verify that elements in the same combination of sets always either belong or do not belong to the same side of the identity. Use 1 to indicate it is in the set and a 0 to indicate that it is not.
Proof of Second De Morgan Law

These steps show that: $A \cap B \subseteq \overline{A} \cup \overline{B}$

$x \in A \cap B$ by assumption
$x \notin A \cap B$
$\neg((x \in A) \land (x \in B))$ defn. of complement
$\neg(x \in A) \lor \neg(x \in B)$ defn. of intersection
$\neg(x \in A) \lor \neg(x \in B)$ 1st De Morgan Law for Prop Logic
$x \notin A \lor x \notin B$ defn. of negation
$x \in \overline{A} \lor x \in \overline{B}$ defn. of complement
$x \in \overline{A} \cup \overline{B}$ defn. of union

Continued on next slide →
Proof of Second De Morgan Law

These steps show that:

\[ A \cup B \subseteq A \cap B \]

- \( x \in A \cup B \) by assumption
- \( (x \in A) \lor (x \in B) \) defn. of union
- \( (x \notin A) \lor (x \notin B) \) defn. of complement
- \( \neg(x \in A) \lor \neg(x \in B) \) defn. of negation
- \( \neg((x \in A) \land (x \in B)) \) by 1st De Morgan Law for Prop Logic
- \( \neg(x \in A \cap B) \) defn. of intersection
- \( x \in A \cap B \) defn. of complement
Set-Builder Notation: Second De Morgan Law

\[
\overline{A \cap B} = \{ x \mid x \notin A \cap B \} \quad \text{by defn. of complement}
\]

\[
= \{ x \mid \neg(x \in (A \cap B)) \} \quad \text{by defn. of does not belong symbol}
\]

\[
= \{ x \mid \neg(x \in A \land x \in B) \} \quad \text{by defn. of intersection}
\]

\[
= \{ x \mid \neg(x \in A) \lor \neg(x \in B) \} \quad \text{by 1st De Morgan law for Prop Logic}
\]

\[
= \{ x \mid x \notin A \lor x \notin B \} \quad \text{by defn. of not belong symbol}
\]

\[
= \{ x \mid x \in \overline{A} \lor x \in \overline{B} \} \quad \text{by defn. of complement}
\]

\[
= \{ x \mid x \in \overline{A \cup B} \} \quad \text{by defn. of union}
\]

\[
= \overline{A \cup B} \quad \text{by meaning of notation}
\]
### Example:
Construct a membership table to show that the distributive law holds.

### Solution:

\[ A \cup (B \cap C) = (A \cup B) \cap (A \cup C) \]

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>B \cap C</th>
<th>A \cup (B \cap C)</th>
<th>A \cup B</th>
<th>A \cup C</th>
<th>(A \cup B) \cap (A \cup C)</th>
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Homework: Page 136

6, 8, 10, 12, 15 (must)

21, 23, 27 (optional)

35, 40 (bonus scores)