使用主成分进行图像描述

一、基础知识

$$x=\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

$$m=E\{x\} \qquad m_x=\frac{1}{K}\sum_{k=1}^{K}x_k$$

$$C_x=E\left\{(x-m_x)(x-m_x)^T\right\}$$

$$C_x=\frac{1}{K}\sum_{k=1}^{K}x_k x_k^T-m_x m_x^T$$

二、霍特林变换

C_x是实对称的，总可以找到一组n个正交的特征向量及其对应的特征值。令A为一个矩阵，其行由C_x的特征向量组成，第一行对应于最大特征值的特征向量，最后一行对应于最小特征值的特征向量。

$$y = A(x - m_x)$$

性质：（1）y向量的均值为零向量。（2） C_x和C_y具有相同的特征值。（3）x可由y重建。 $x = A^{-1}y + m_x$

$$x = A^T y + m_x$$

酉矩阵：$A^{-1} = A^T$

三、霍特林变换的几何意义

$$y = A(x - m_x)$$

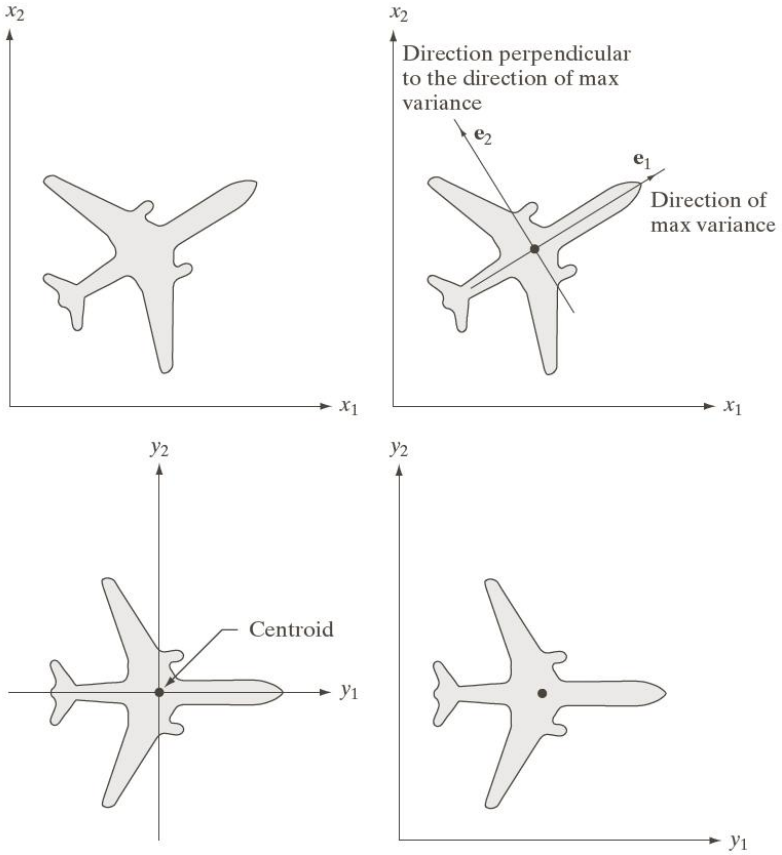
$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\mathbf{m_x} = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$$

$$\mathbf{C_x} = \begin{bmatrix} 3.333 & 2.00 \\ 2.00 & 3.333 \end{bmatrix}$$

$$\mathbf{e}_1 = \begin{bmatrix} 0.707 \\ 0.707 \end{bmatrix}$$

$$\mathbf{e}_2 = \begin{bmatrix} -0.707 \\ 0.707 \end{bmatrix}$$



a b
c d

FIGURE 11.43
(a) An object. (b) Object showing eigenvectors of its covariance matrix. (c) Transformed object, obtained using Eq. (11.4-6). (d) Object translated so that all its coordinate values are greater than 0.

（1）协方差的最大特征值所确定的特征向量方向，对应于物体的最大方差方向；协方差的最小特征值所确定的特征向量方向，对应于物体的最小方差方向；这些方向相互正交。

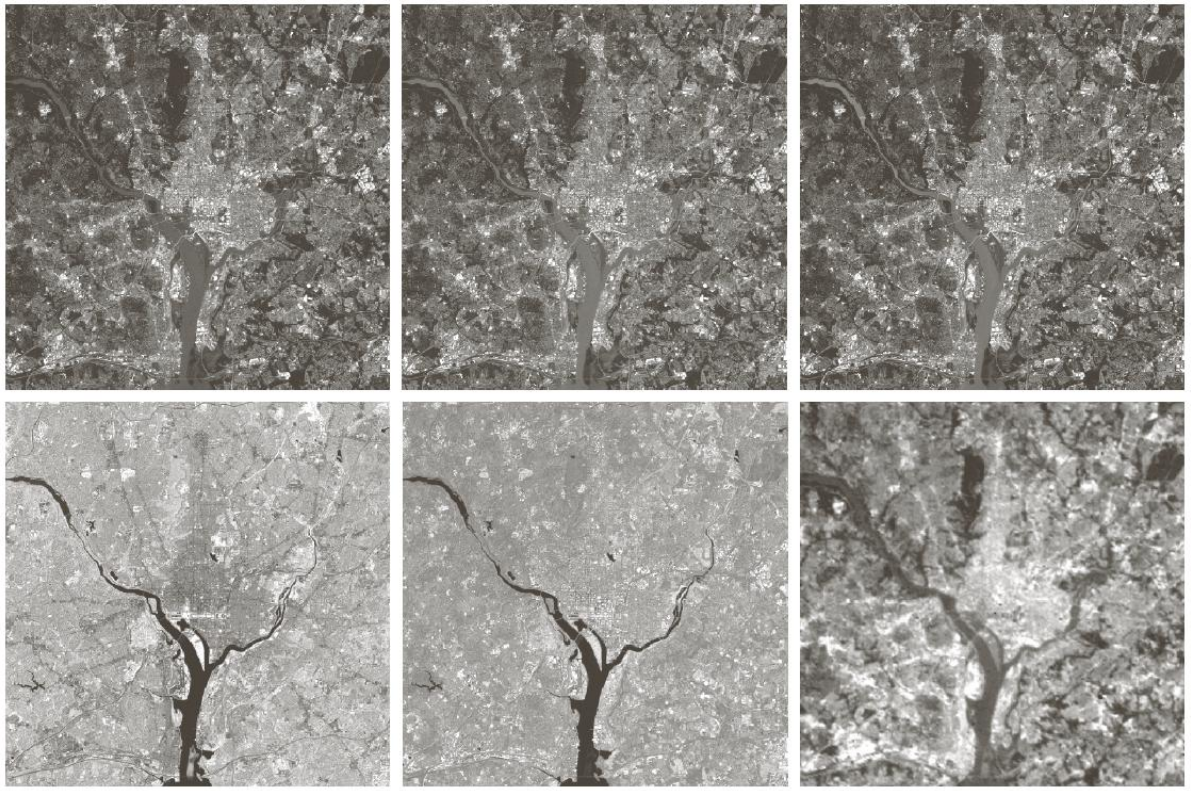
（2）霍特林变换，实质上实现了物体中心原点化及物体方差方向坐标轴化。

四、主成分变换

$$y = A_k\left(x - m_x\right)$$

$$\widehat{x} = A_k^T y + m_x$$

$$e_{ms} = \frac{1}{nn}\sum\left\|x - \widehat{x}\right\|_2 = \sum_{j=1}^{n}\lambda_j - \sum_{j=1}^{k}\lambda_j = \sum_{j=k+1}^{n}\lambda_j$$



a b c
d e f

FIGURE 11.38 Multispectral images in the (a) visible blue, (b) visible green, (c) visible red, (d) near infrared, (e) middle infrared, and (f) thermal infrared bands. (Images courtesy of NASA.)

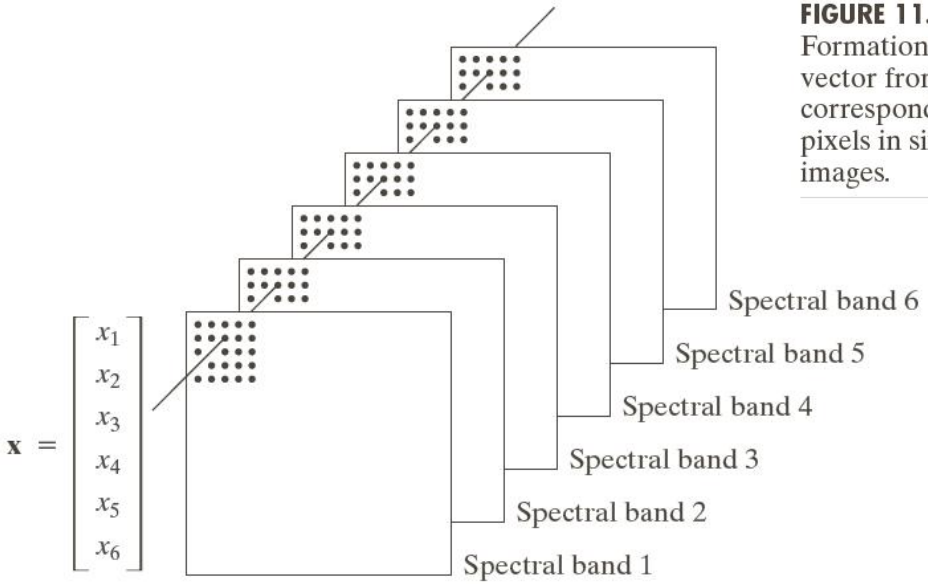
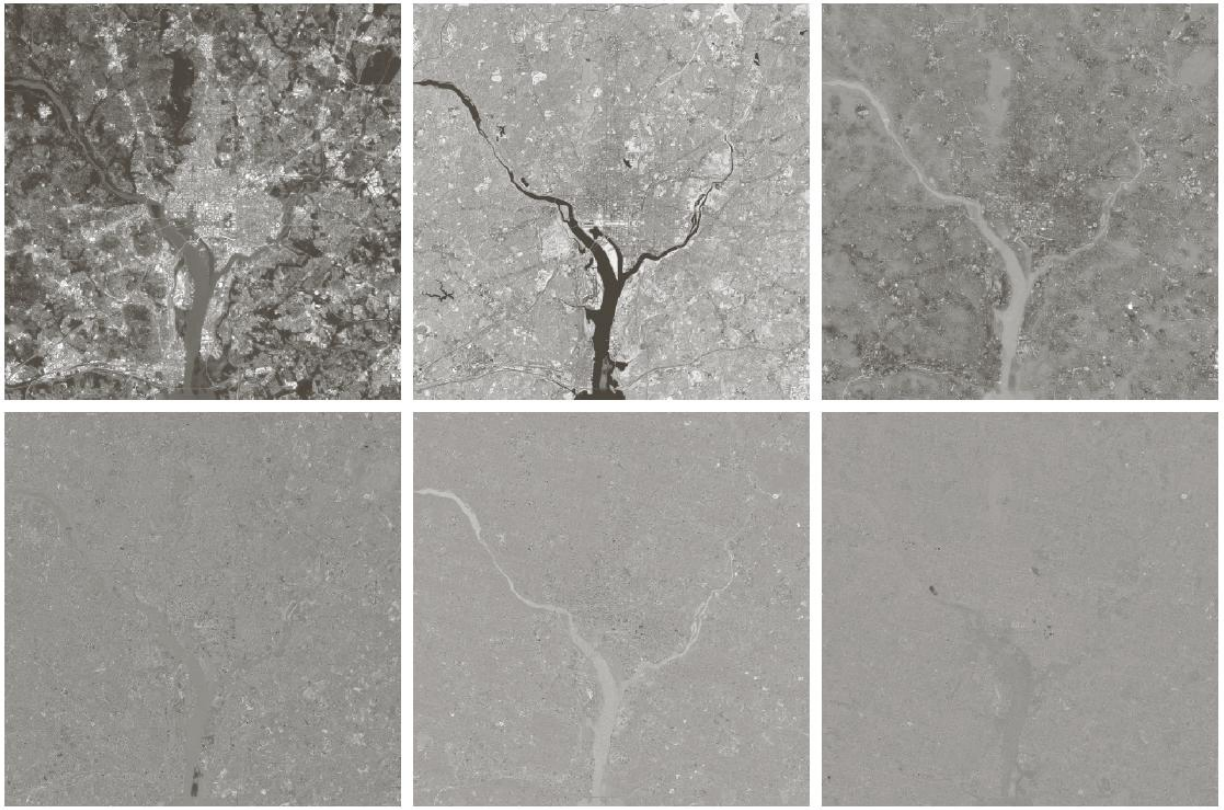


FIGURE 11.39
Formation of a vector from corresponding pixels in six images.

λ_1	λ_2	λ_3	λ_4	λ_5	λ_6
10344	2966	1401	203	94	31

TABLE 11.6
Eigenvalues of the covariance matrices obtained from the images in Fig. 11.38.

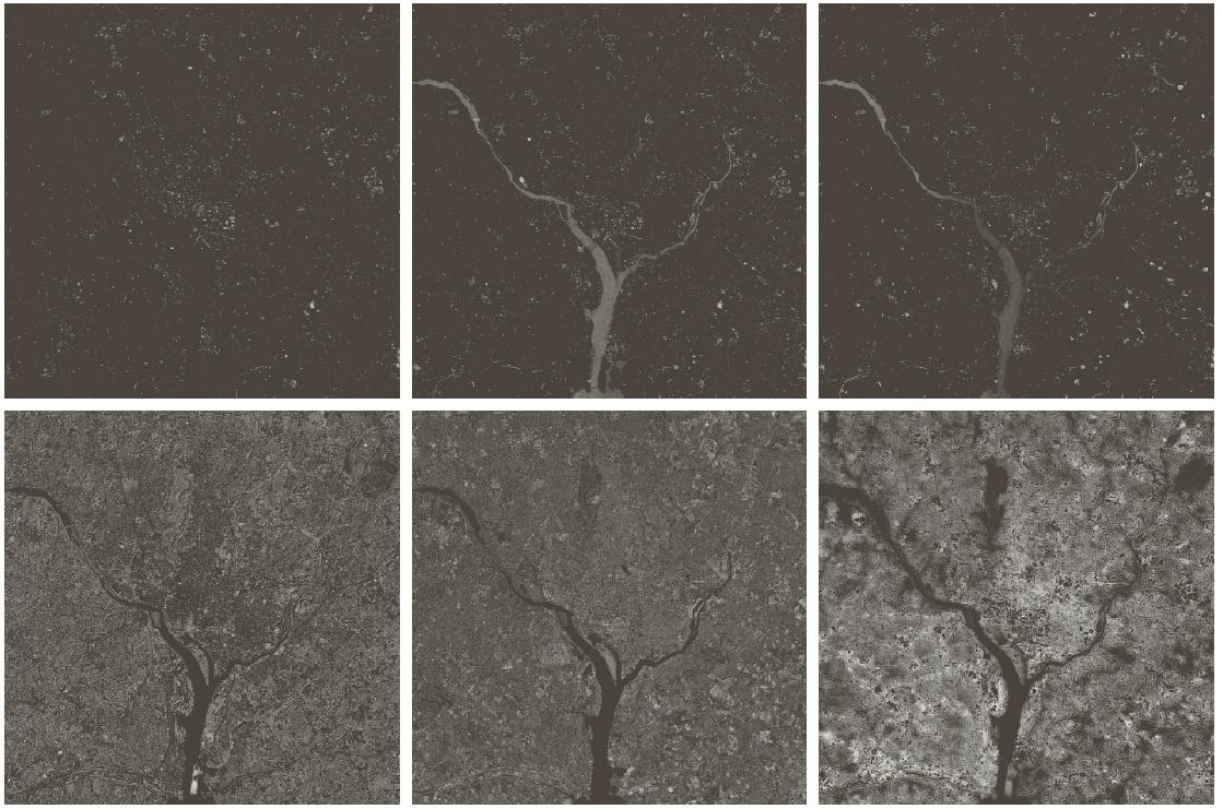


a b c
d e f

FIGURE 11.40 The six principal component images obtained from vectors computed using Eq. (11.4-6). Vectors are converted to images by applying Fig. 11.39 in reverse.

特征压缩

图像融合



a b c
d e f

FIGURE 11.42 Differences between the original and reconstructed images. All difference images were enhanced by scaling them to the full [0, 255] range to facilitate visual analysis.

由前两个成分重建

休 息 一 下